IstitutoVeneto diScienzeLettere edArti

SUMMER SCHOOL ON BIOGEODYNAMICS AND EARTH SYSTEM SCIENCES

BESS

Boolean Delay Equations: Theory and Insights into the El Nino Southern Oscillation

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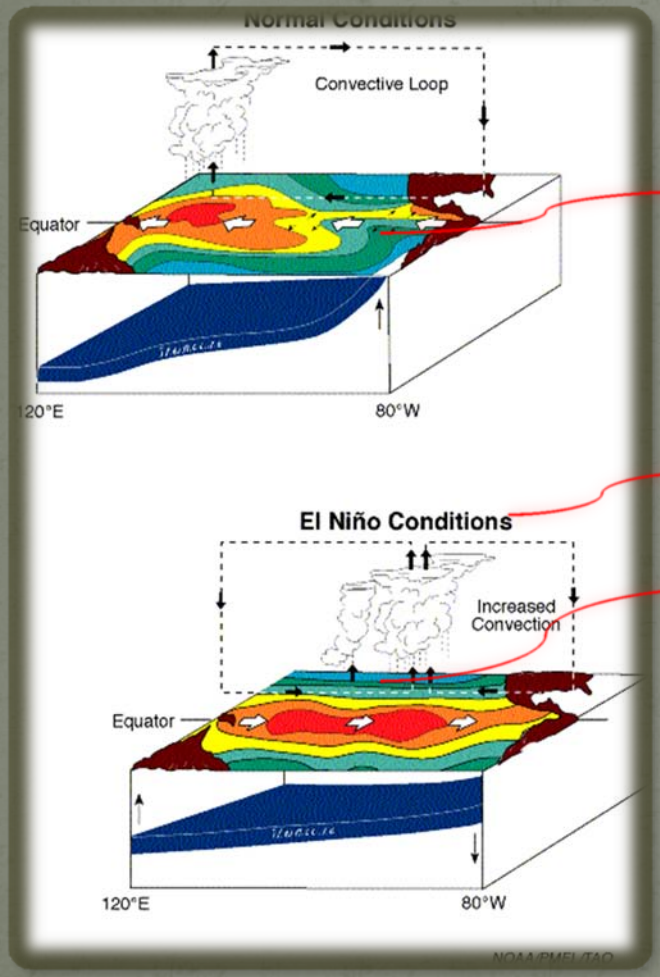
The El Nino-Southern Oscillation

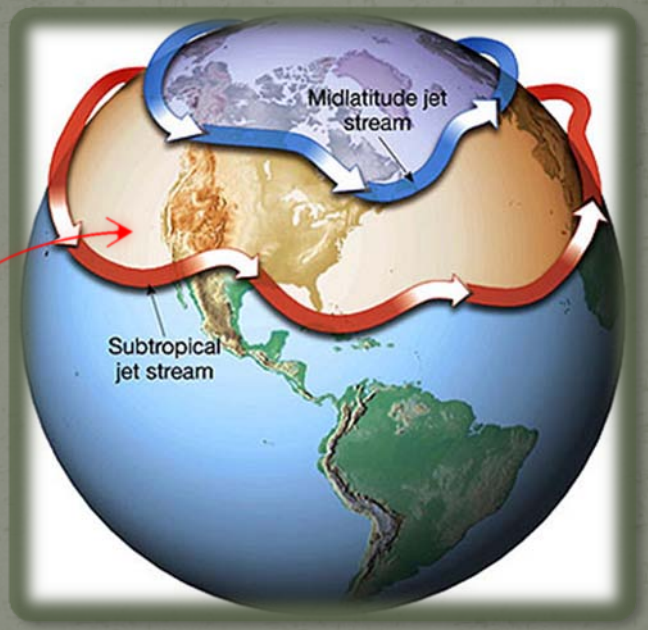


Rossby waves

U_3

T_1, T_2

U_1, U_2



Rossby wave travel time: τ=0.4-2.0

Local delays: β=0.2

Period of mid-latitude jet: θ=0.05-0.2

Response of tropics to mid-lat forcing: γ=0.05-0.1

The ENSO model

$$U_1(t) = T_1(t-\beta)$$

$$U_2(t) = T_2(t-\beta)$$

$$R(t) = U_1(t)\Delta U_2(t)$$

$$T_1(t) = \{(R \wedge \neg U_1)(t-\tau)\} \vee \{\neg R(t-\tau) \wedge U_2(t-\beta)\}$$

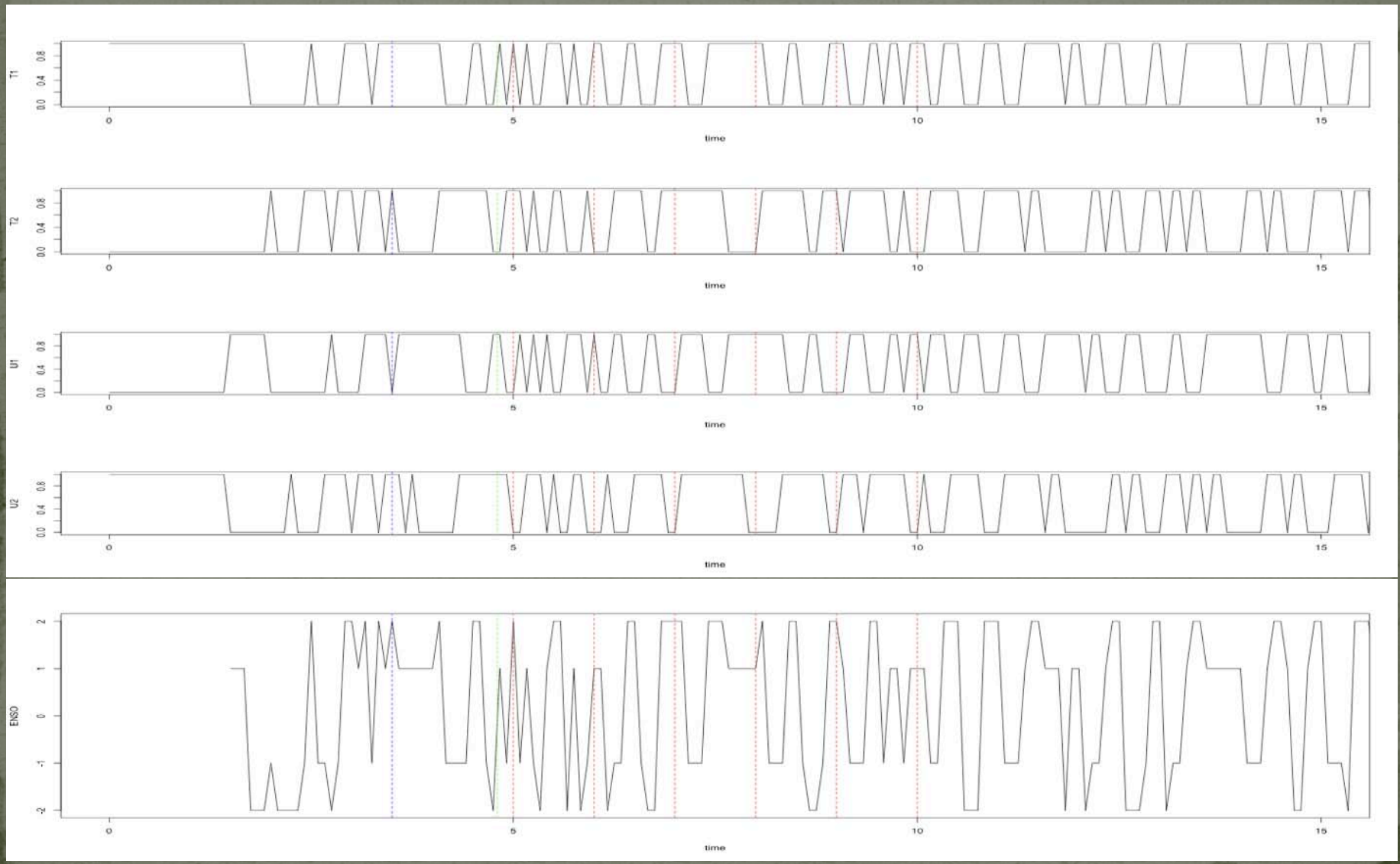
$$T_2(t) = \{(S\Delta T_1)(t-\beta)\}$$

$$S(t) = S(t-1)$$

PHASE (T_1)	INTENSITY (T_2)	ENSO	
0	0	-2	Strong La Niña
0	1	-1	Mild La Niña
1	0	1	Mild El Niño
1	1	2	Strong El Niño

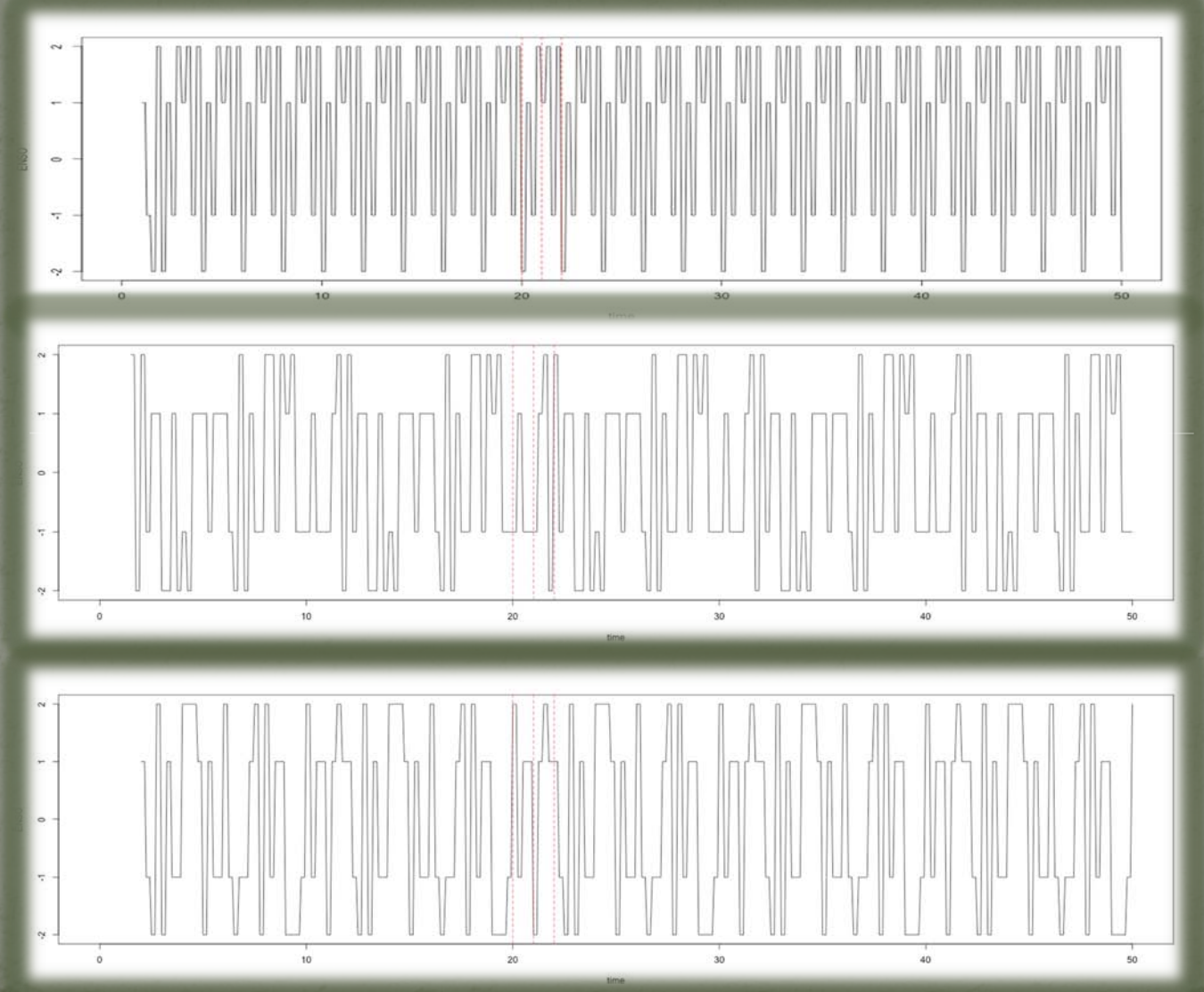
DIRECTION (U_1)	INTENSITY (U_2)	
0	0	Extreme easterly anomalies
0	1	Mild easterly anomalies
1	0	Mild westerly anomalies
1	1	Extreme westerly anomalies

Example of the march of phases



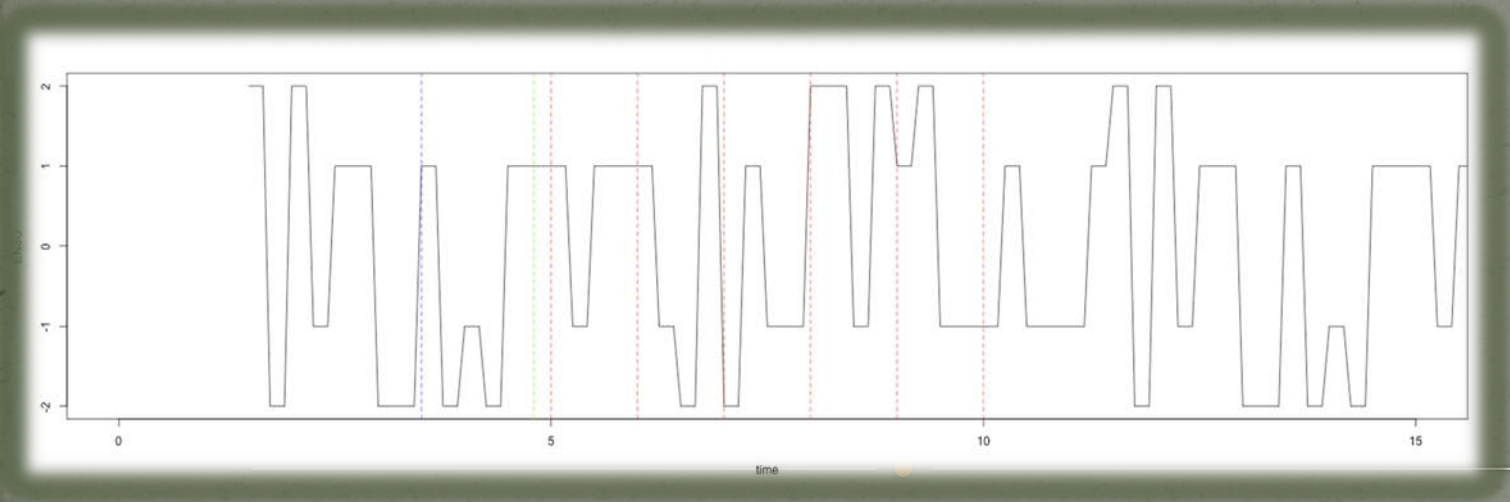
Sensitivity to the return period of Rossby waves

Initial state: mild warm, 6 months winter

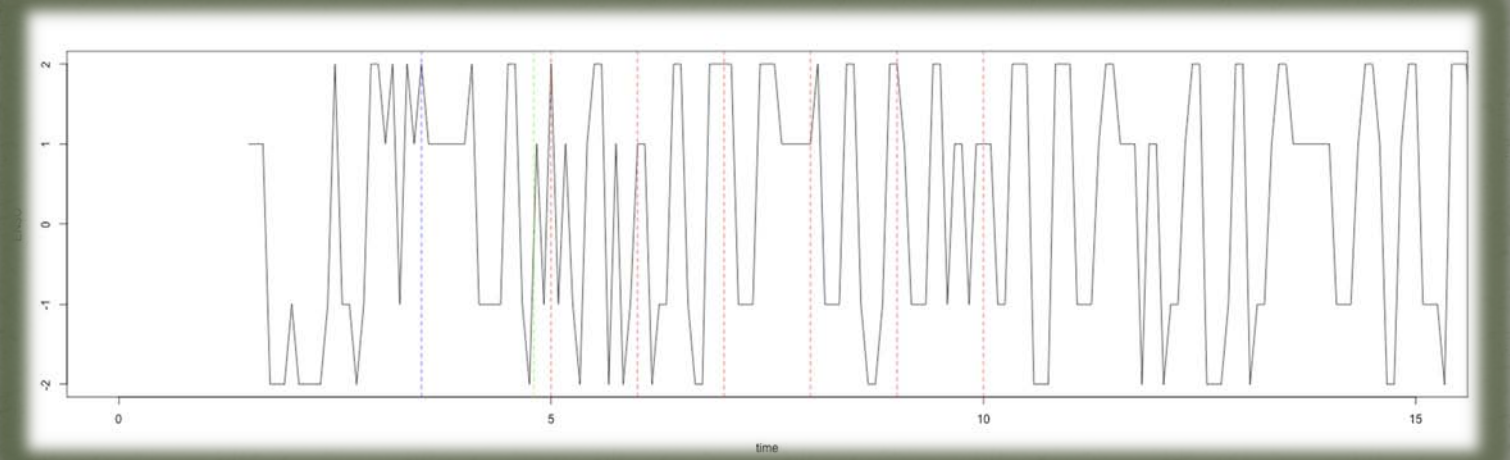


Sensitivity to seasonality

Initial state: mild warm, τ-1.5



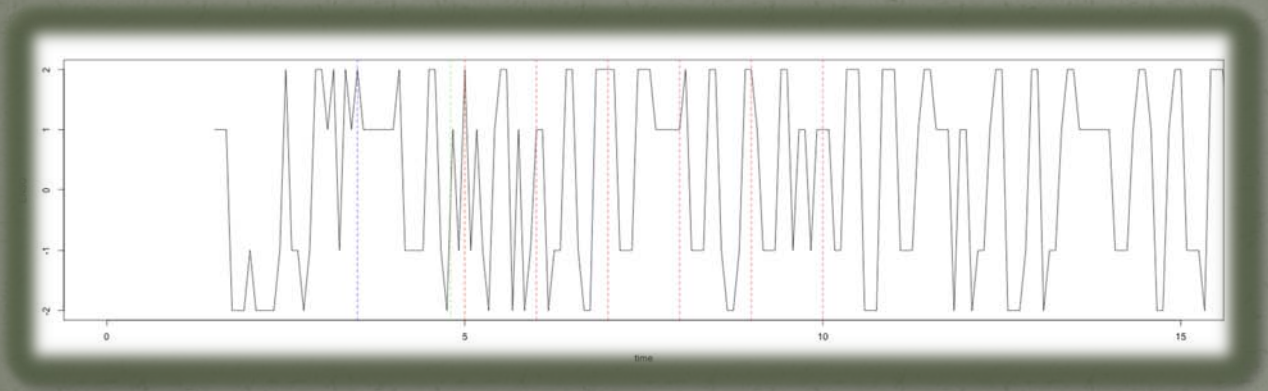
Half-yr winter



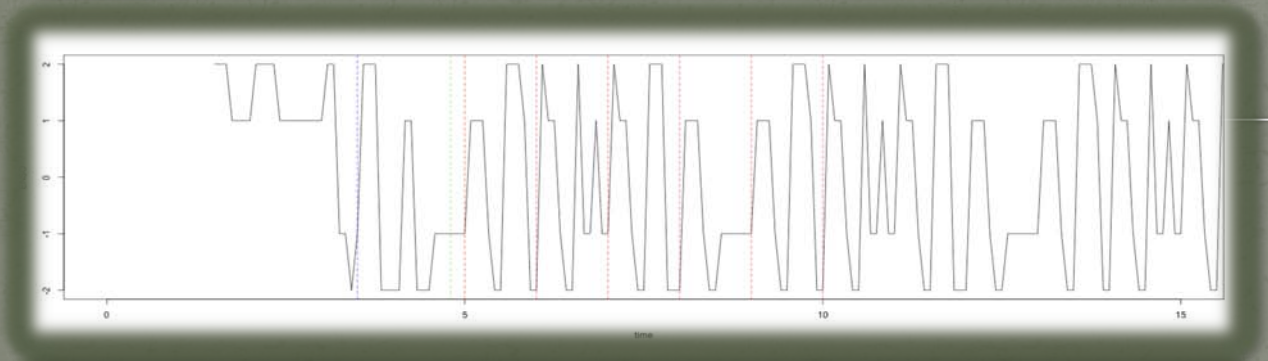
NDJF winter

Sensitivity to the initial state

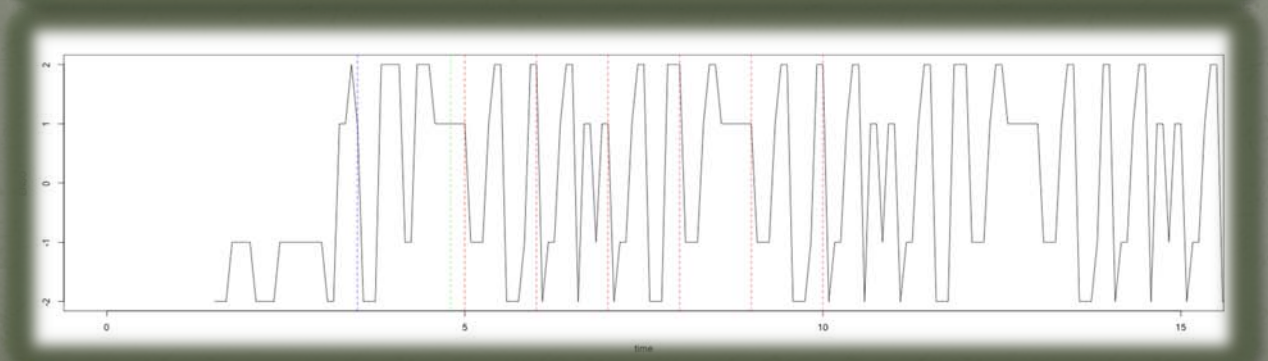
Initial state: NDJF winter, τ-1.5



Mild warm conditions



Strong El Nino



Strong La Nina

First, some theory… how to model a driving factor

$$(1)\quad \begin{cases} x(t) = \neg x(t-1) \\ y(t) = \neg y(t-1) \vee \neg x(t-\theta) \end{cases}$$

via coupling with an additional BDE

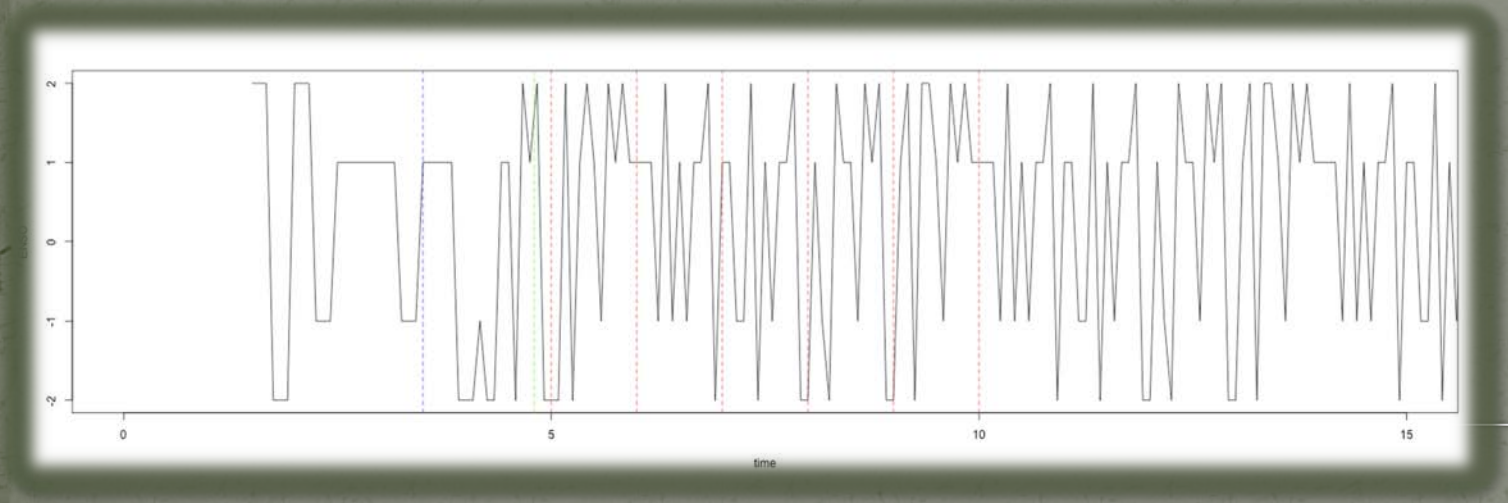
Coupling ENSO and mid-latitude circulation:

$$(2)\quad \begin{cases} x(t) = \neg x(t-1) \\ U_1(t) = T_1(t-\beta) \vee \neg U_3(t-\gamma) \\ y(t) = \neg y(t-\tau) \\ \tau = \begin{cases} \tau_1 & x = 0 \\ \tau_2 & x = 1 \end{cases} \end{cases}$$

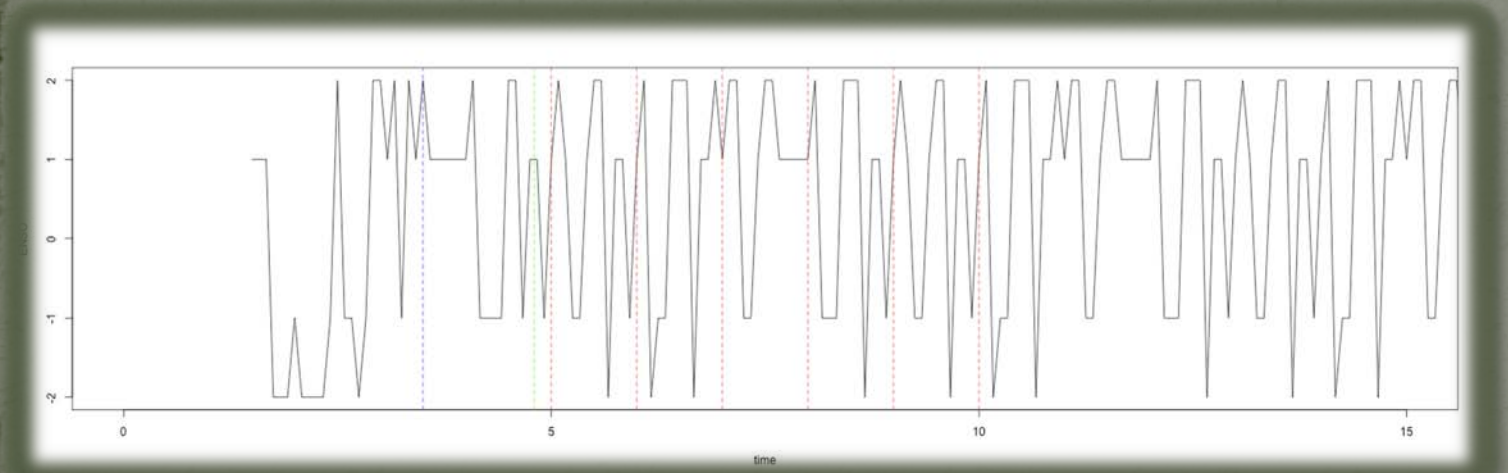
via state-dependent delay

Sensitivity to seasonality

Initial state: mild warm, τ-1.5



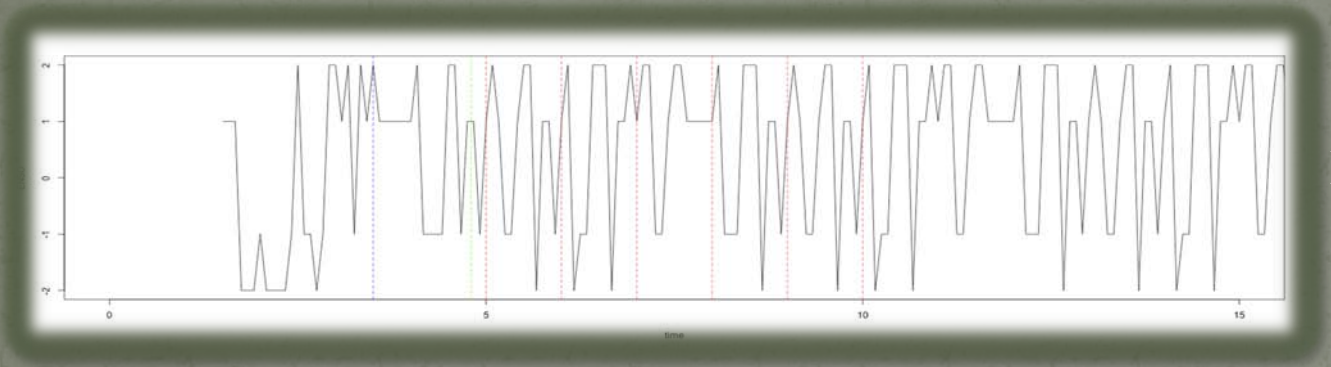
Half-yr winter



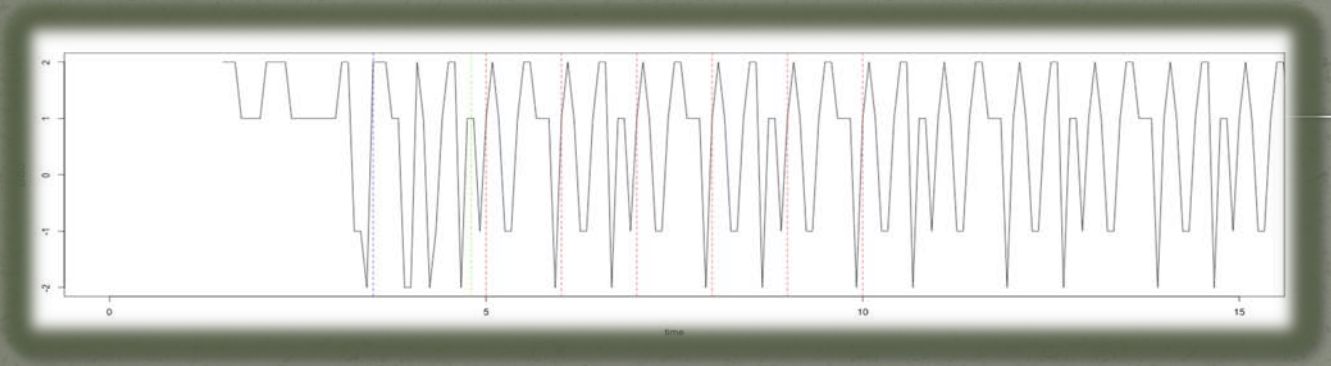
NDJF winter

Sensitivity to the initial state

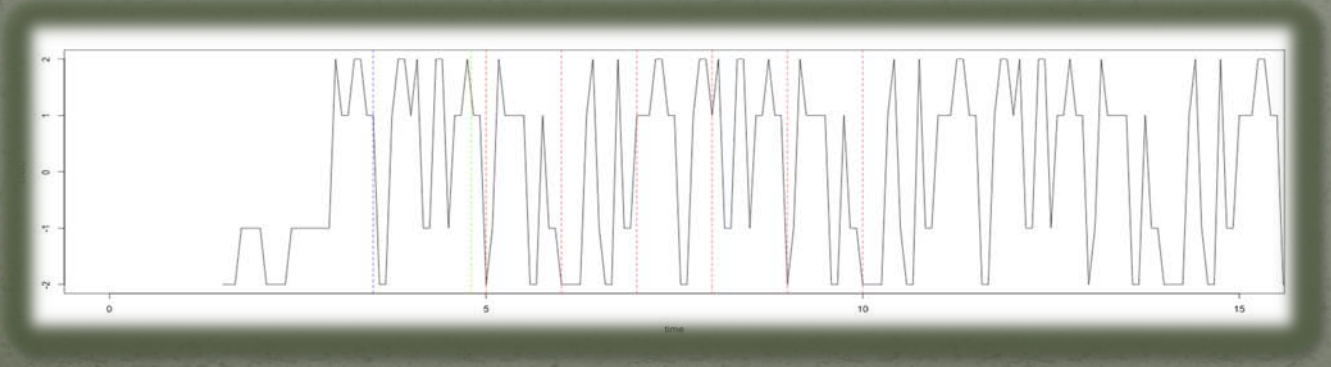
Initial state: NDJF winter, τ=1.5



Mild warm conditions



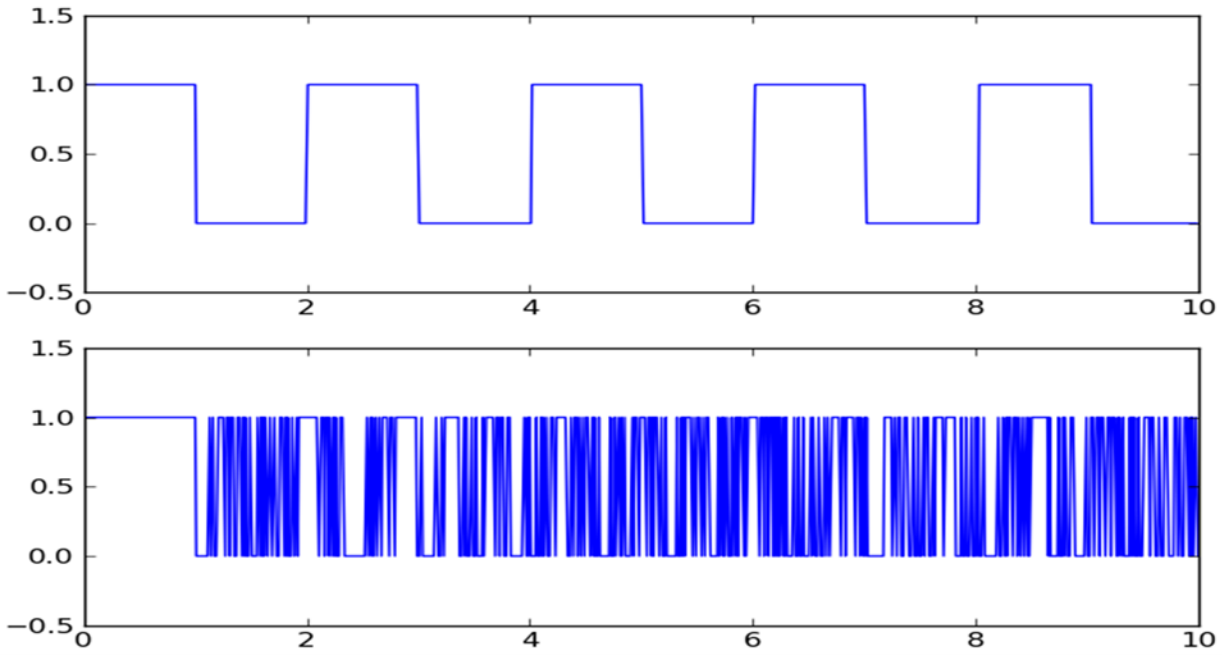
Strong El Nino



Strong La Nina

BDEs with random delay

$$\begin{cases} x(t) = \neg x(t-1) \\ y(t) = \neg y(t-\tau) \end{cases} \quad if \quad x = \begin{cases} 0 & \tau \in P_0(\tau) \\ 1 & \tau \in P_1(\tau) \end{cases} \qquad P_j(\tau) = \mu_j + \mathrm{N}(0,\sigma)$$



Quiz…

1) Do you expect to see more 0's or more 1's?

Ok, that was easy! But, why??

2) How many jumps do you expect to see?

3) Each time you are in state 1 (or state 0), how long do you expect to stay in the same state?

$$\mu_0 = 0.1$$
$$\mu_1 = 0.2$$

t	0	1
10	451	447
100	4906	4932
1000	49939	49960

1)

$$\mu_0 = 0.2$$
$$\mu_1 = 0.4$$

t	0	1
10	449	450
100	4939	4960
1000	49946	49953

Quiz…

1) Do you expect to see more 0's or more 1's?

2) How many jumps do you expect to see?

3) Each time you are in state 1 (or state 0), how long do you expect to stay in the same state?

$\mu_0 = 0.1$

$\mu_1 = 0.2$

t	# JUMPS
10	408
100	4628

2)

$\mu_0 = 0.2$

$\mu_1 = 0.4$

t	# JUMPS
10	358
100	4364

Quiz…

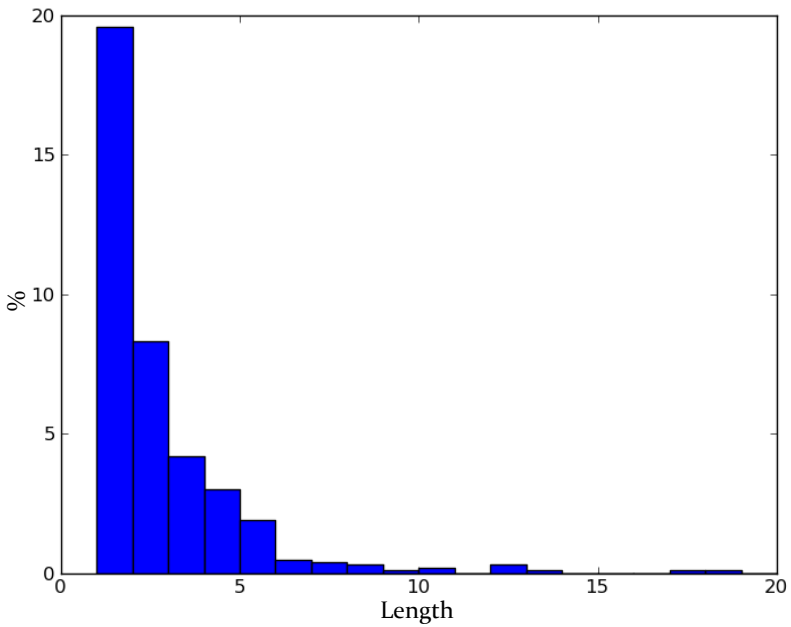
1) Do you expect to see more 0's or more 1's?

2) How many jumps do you expect to see?

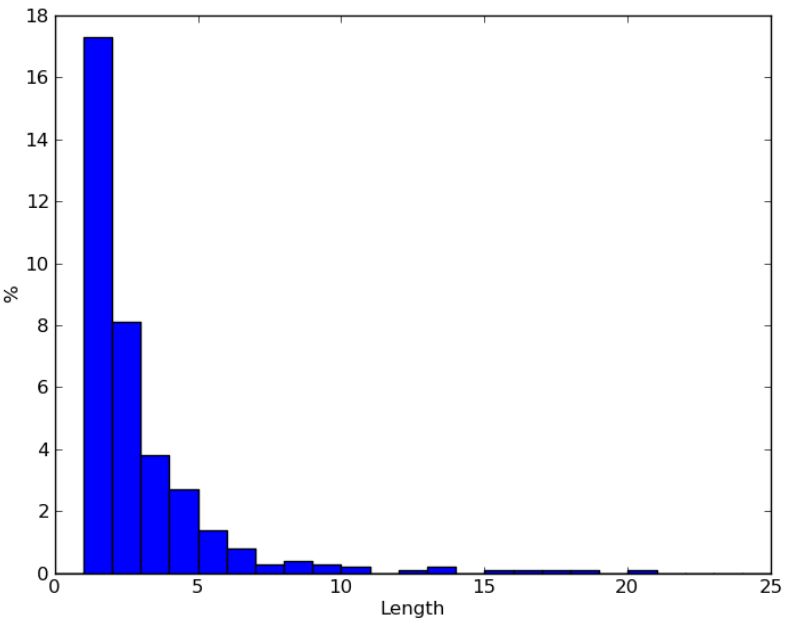
3) Each time you are in state 1 (or state 0), how long do you expect to stay in the same state?

Distribution of the spells of 0 and 1

Dependence on the length of the domain

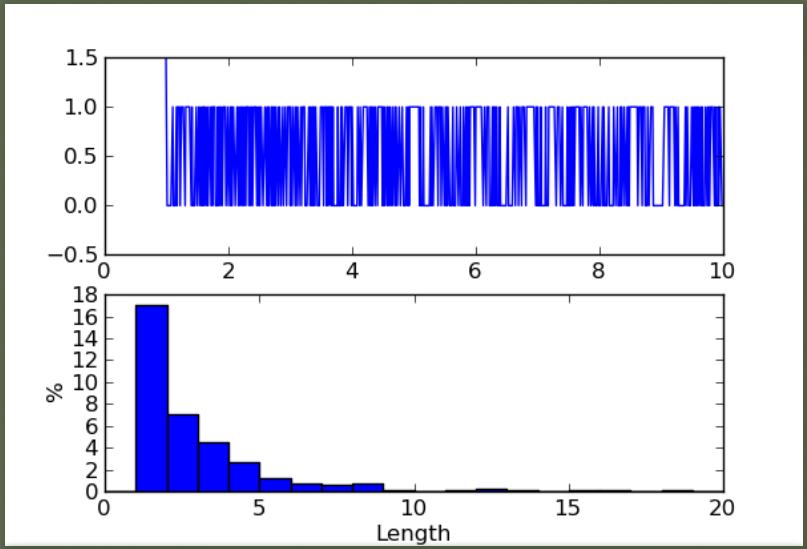


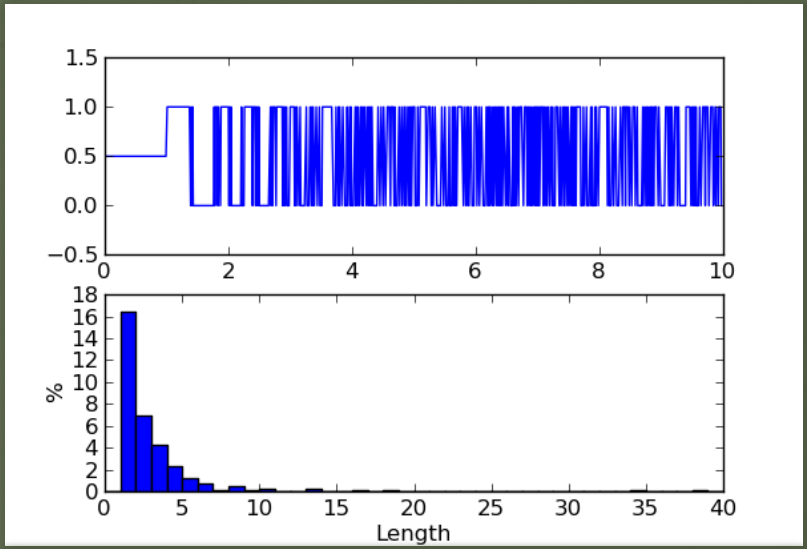
t=10

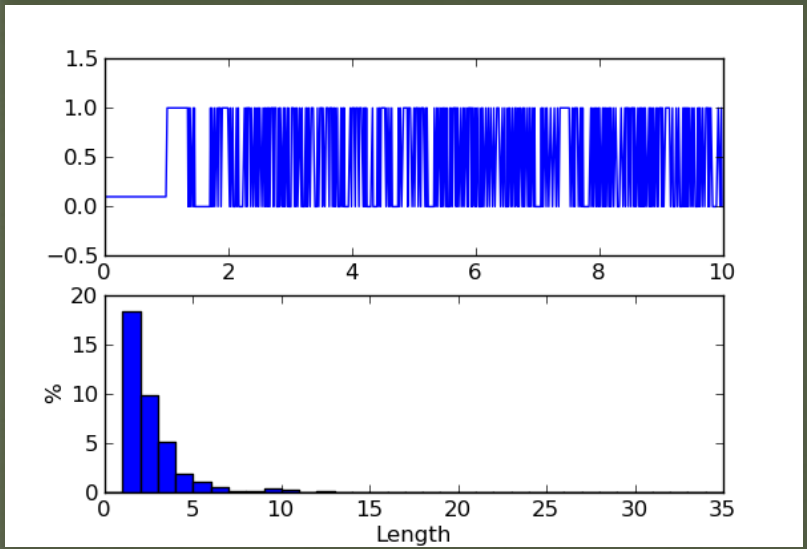


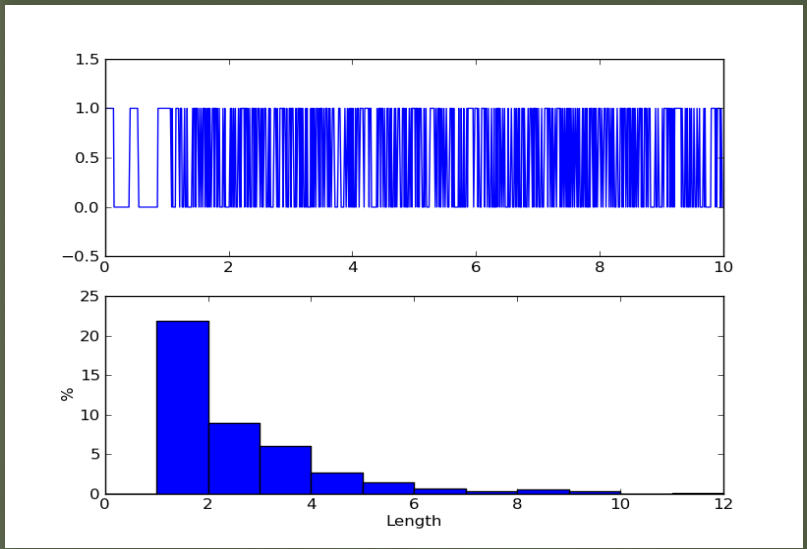
t=100

Dependence on the initial data





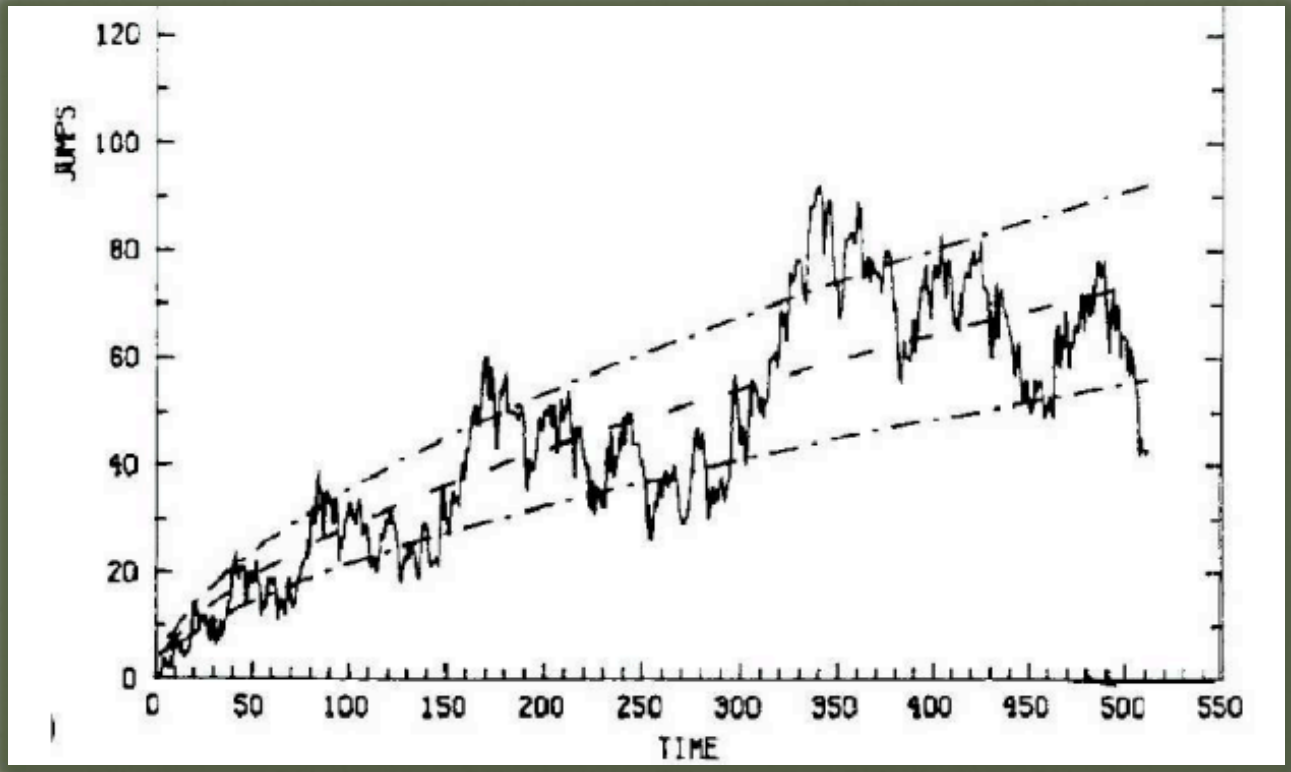




Is it always like this??

$$\begin{cases} x(t) = \neg x(t-\tau) \\ y(t) = y(t-1) \vee y(t-\theta) \end{cases}$$

$$\theta = \frac{\sqrt{5}-1}{2}$$

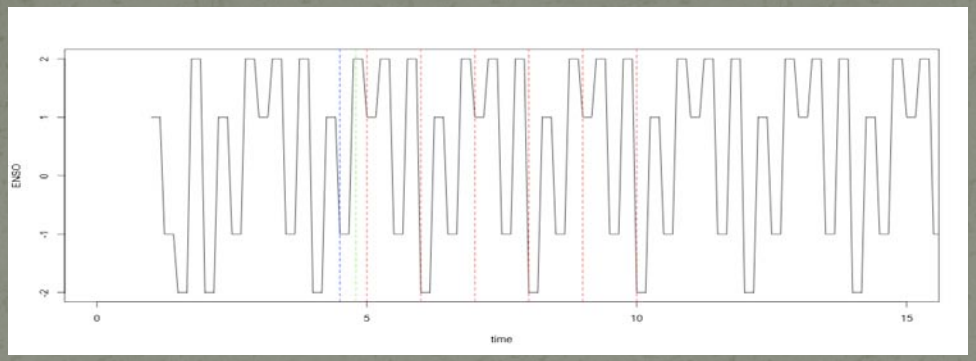


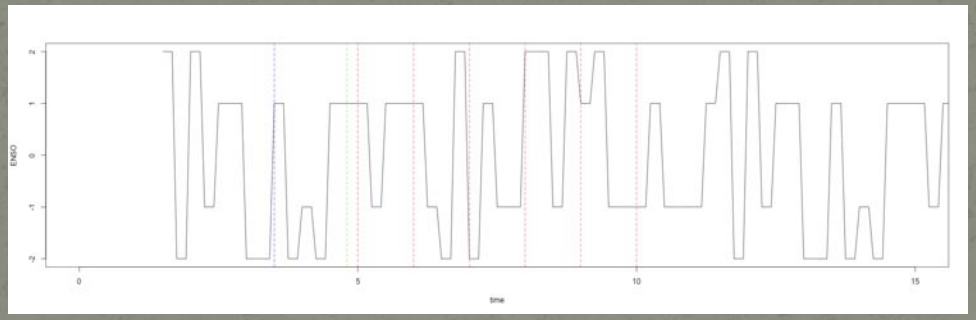
Ghil et al., 2008, 'Boolean Delay Equations: A Simple Way of Looking at Complex Systems'

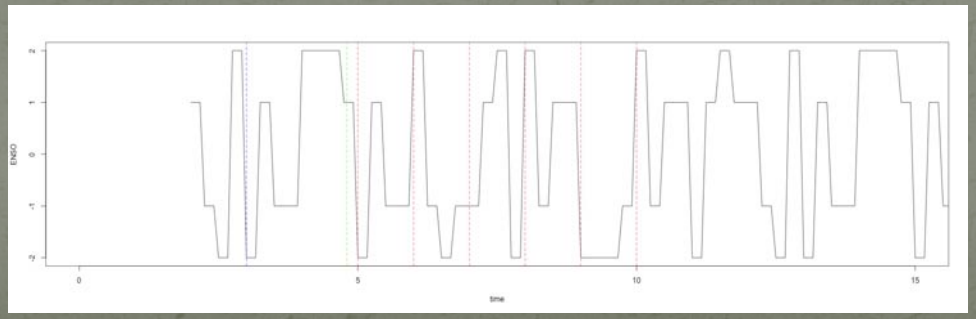
“Fools ignore complexity. Pragmatists suffer it. Some can avoid it. Geniuses remove it.”

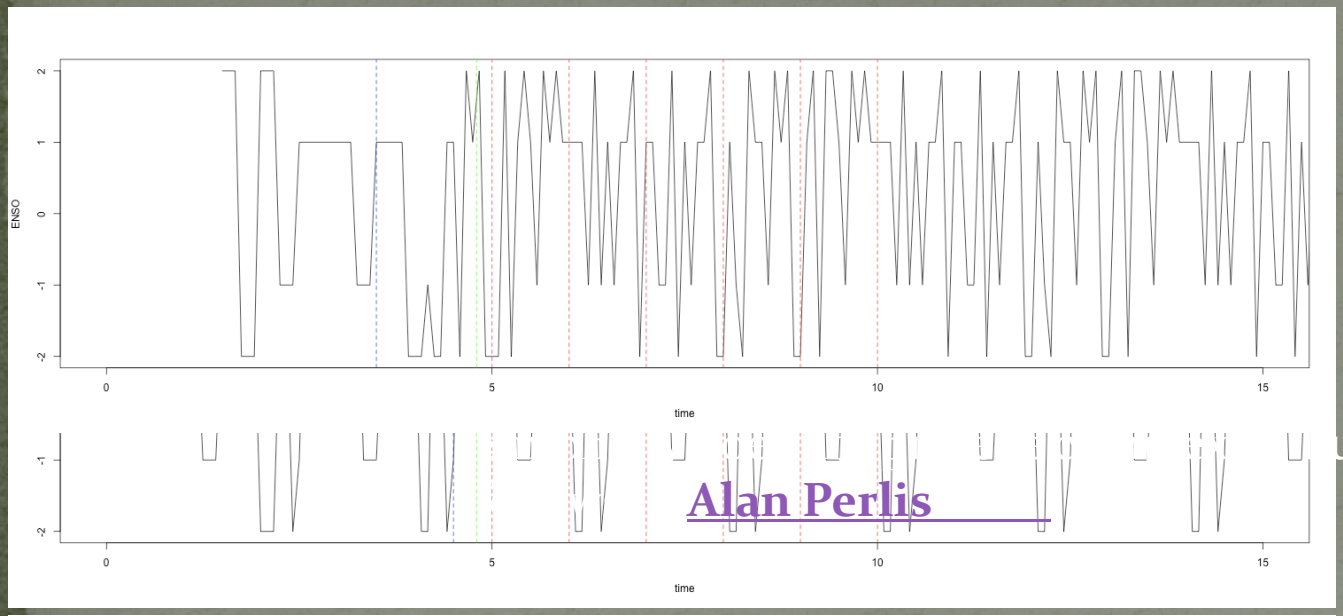
Alan Perlis, 1922-1990

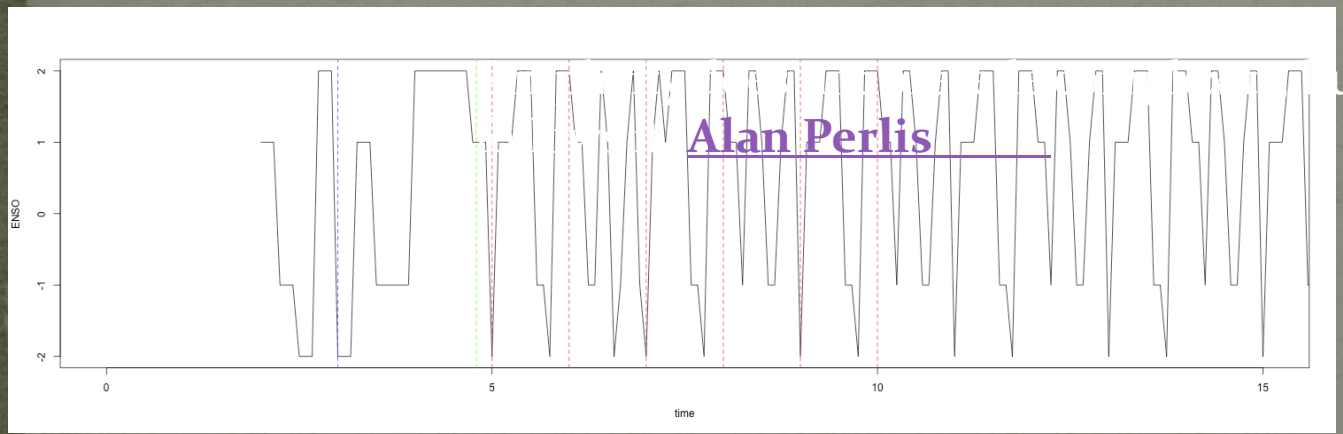
Reserves


ENSO
2
1
0
-1
-2
0
5
10
15
time


ENSO
2
1
0
-1
-2
0
5
10
15
time

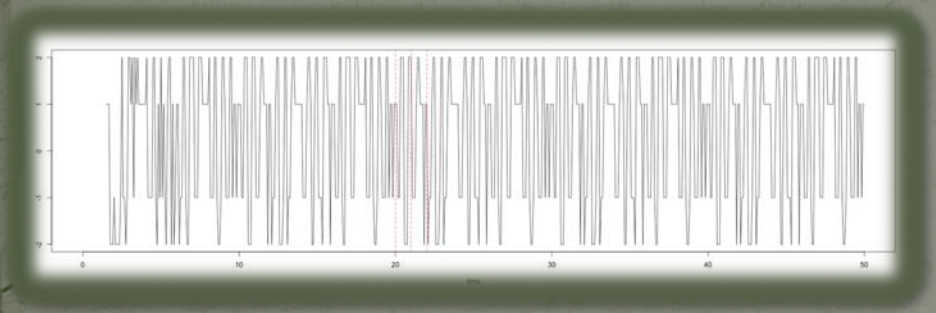

ENSO
2
1
0
-1
-2
0
5
10
15
time

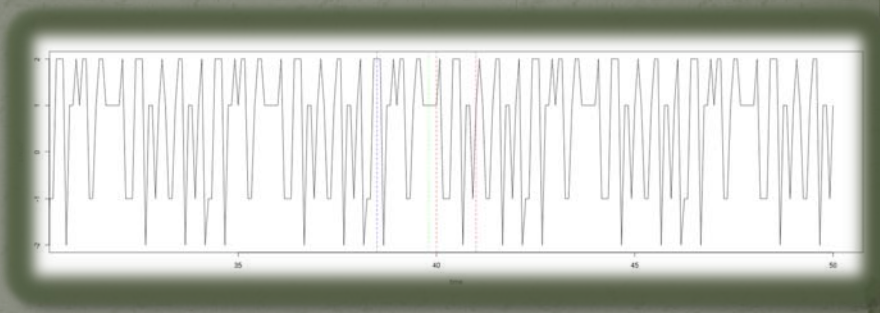

ENSO
time
Alan Perlis

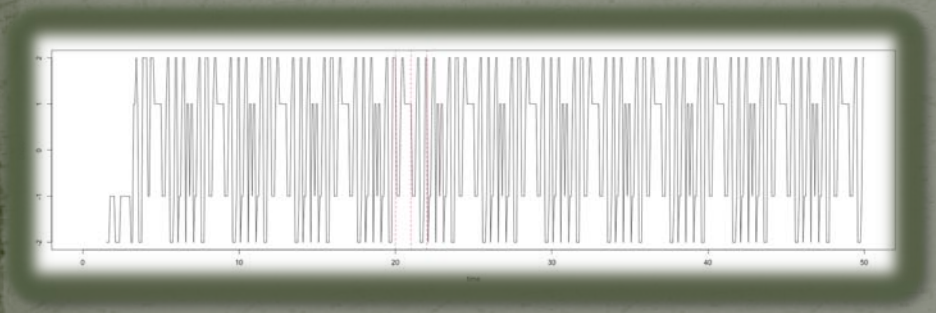

ENSO
time
Alan Perlis

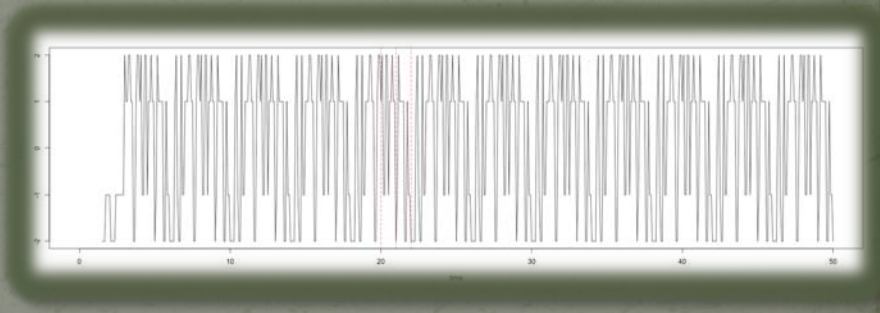
Uncoupled vs. Coupled

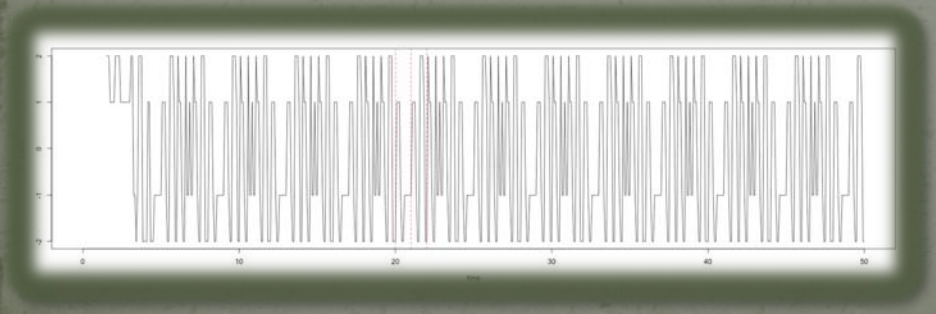
Initial state: NDJF winter, τ=1.5

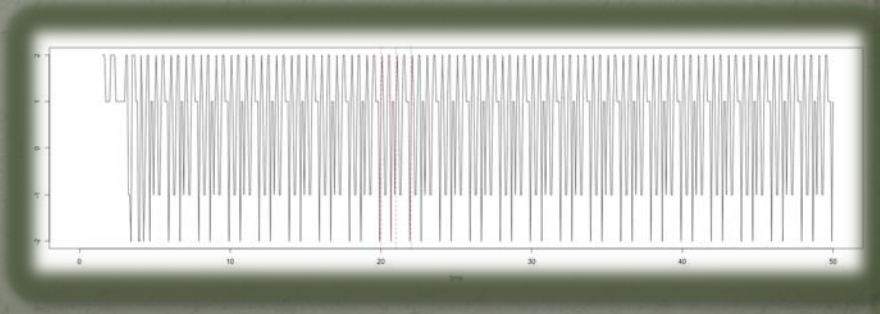












Sensitivity to the return period of Rossby waves

coupled, Initial state: mild warm, 6 months winter