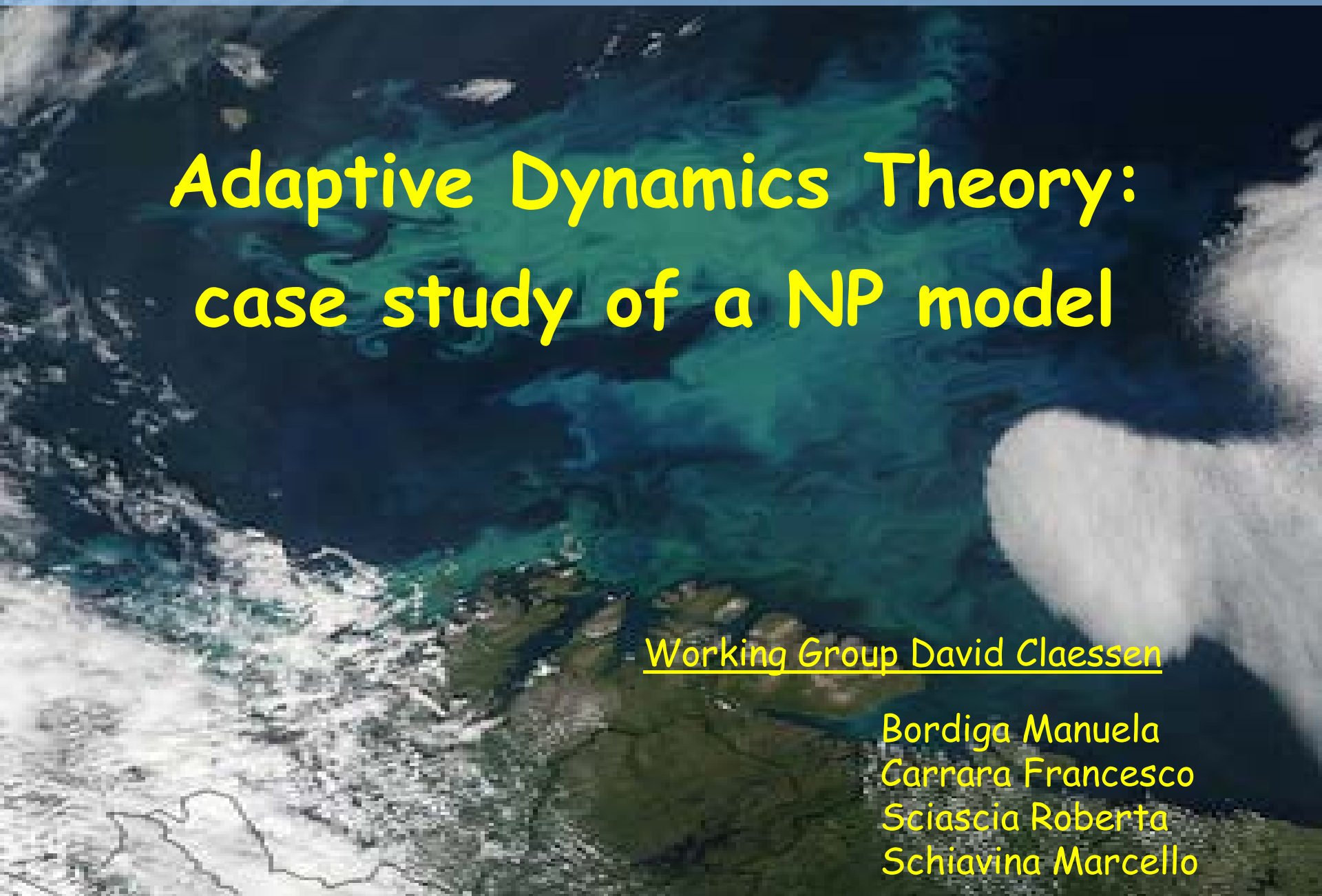


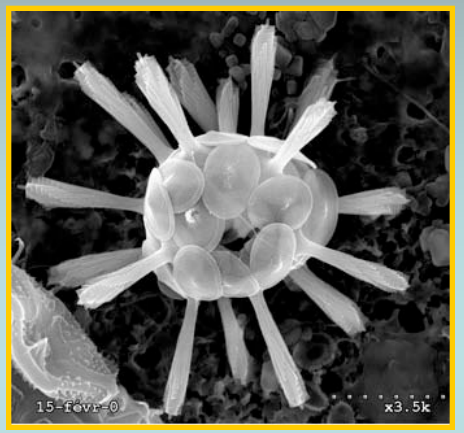
Adaptive Dynamics Theory: case study of a NP model

Working Group David Claessen

Bordiga Manuela
Carrara Francesco
Sciascia Roberta
Schiavina Marcello

OUTLINE

Study the adaptive dynamics of phytoplankton population in a simplified model.



Rhabdosphaera clavigera

- An introduction to the adaptive dynamics theory

- Case study:
  1 – two resources – one consumer model
  2 – predation depending on size
  3 – NPZ model

Centric diatom

ADAPTIVE DYNAMICS

Adaptive dynamics (AD) is a mathematical theory that explicitly links population dynamics to long-term evolution driven by mutation and selection.

The **fitness** of a given type in a given stationary environment can be defined as the exponential growth rate of a population of mutants in that environment.

For **mutants** the environment is set by the population dynamics of the resident type.

Fitness is not a given quantity, but depend on:

- the traits of the individuals;
- the environment in which they live.

PIP: Pairwise Invasibility Plot

Evolutionary progress is determined by the sign and the size of the fitness of potential mutants.

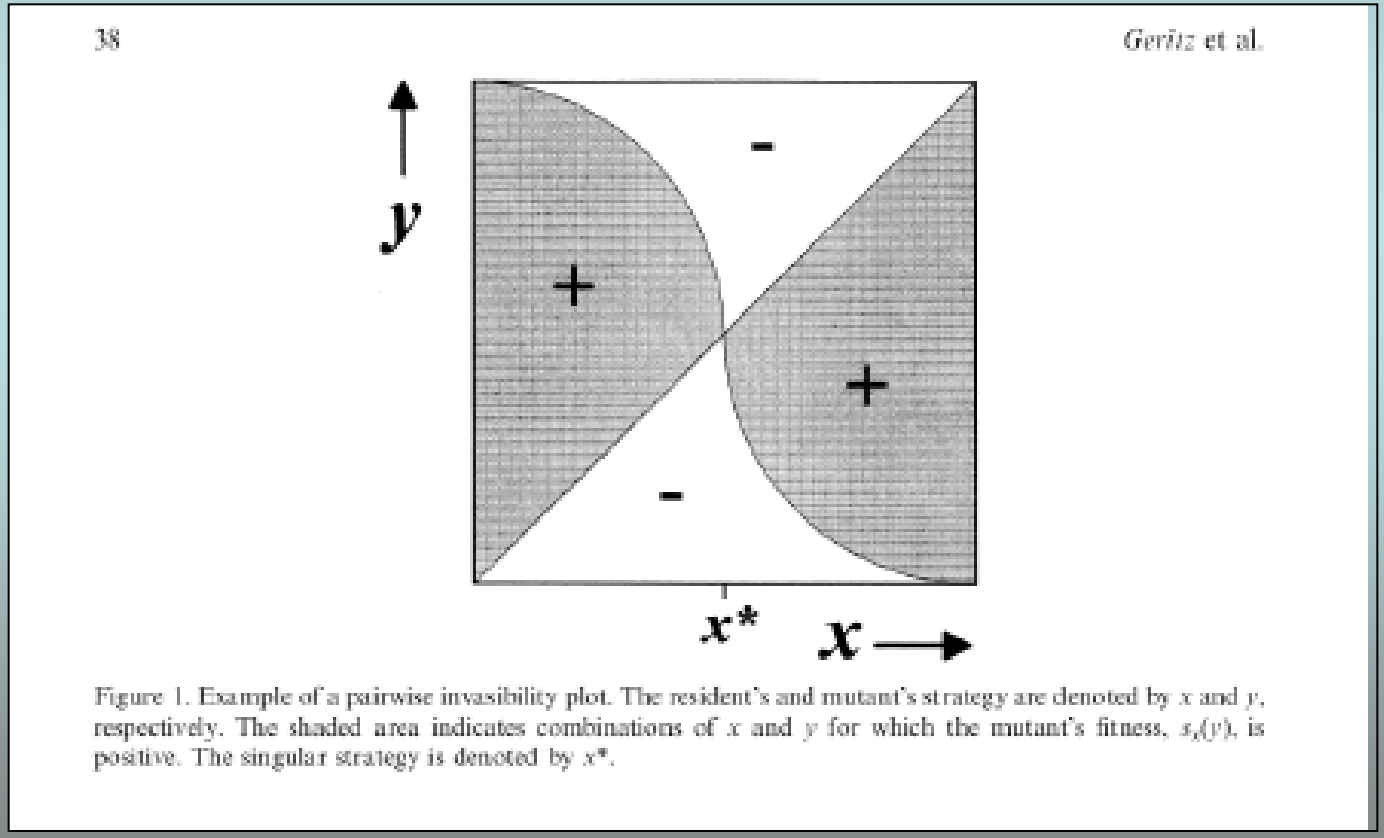


Figure 1. Example of a pairwise invasibility plot. The resident's and mutant's strategy are denoted by x and y, respectively. The shaded area indicates combinations of x and y for which the mutant's fitness, $s_x(y)$, is positive. The singular strategy is denoted by x^*.

NP MODEL

$$\begin{cases} \dfrac{\mathrm{d}N}{\mathrm{d}t} = I - g(N,L)\mu(x)P \\ \dfrac{\mathrm{d}P}{\mathrm{d}t} = [g(N,L)\mu(x) - m - s]P \end{cases}$$

Nutrient

Consumption rate

Phytoplankton

Growth rate

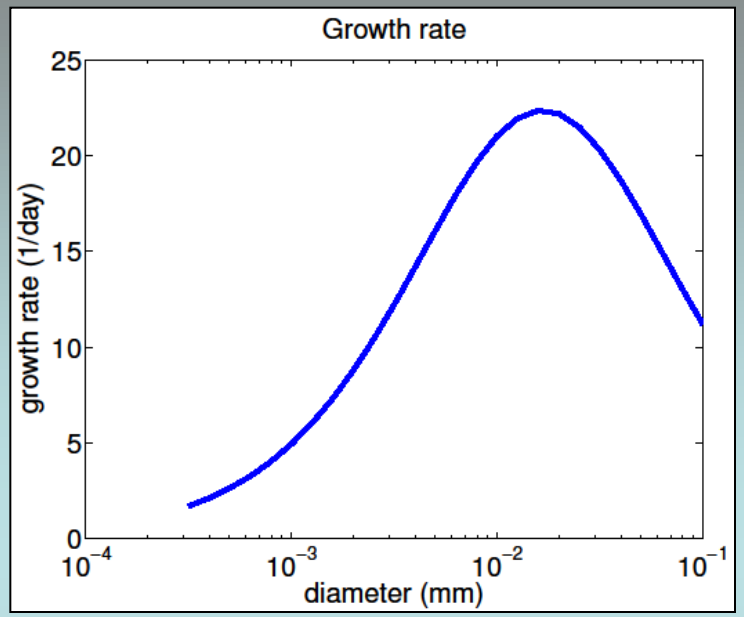
Mortality

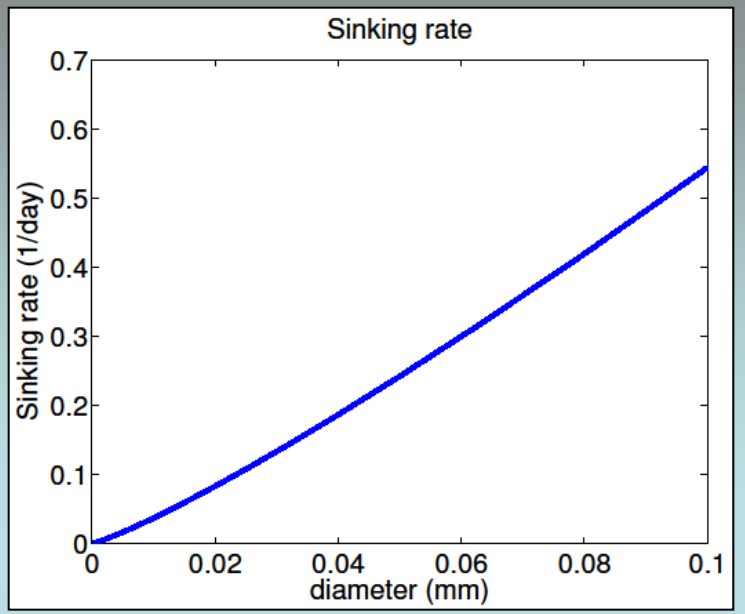
Sinking

Nutrient → PO_4 and NH_4

$$\mu(x) = \frac{x}{ax^2 + bx + c}$$

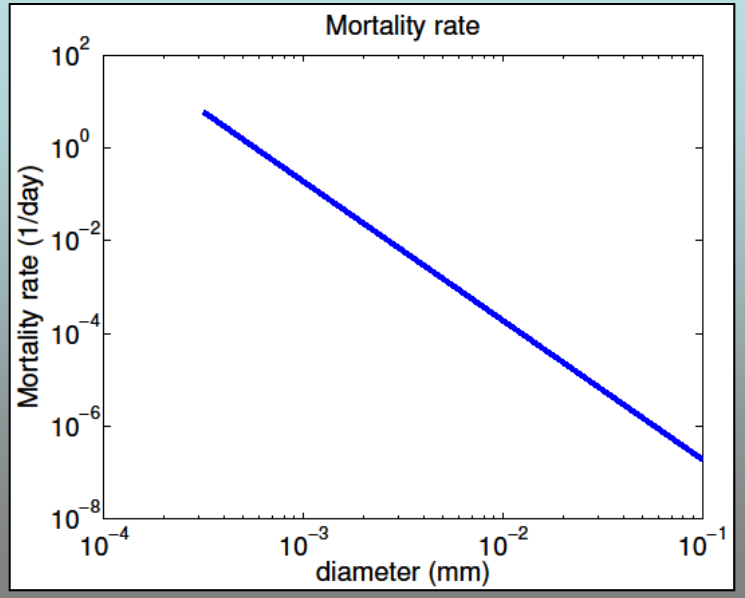
NP MODEL - Parameters



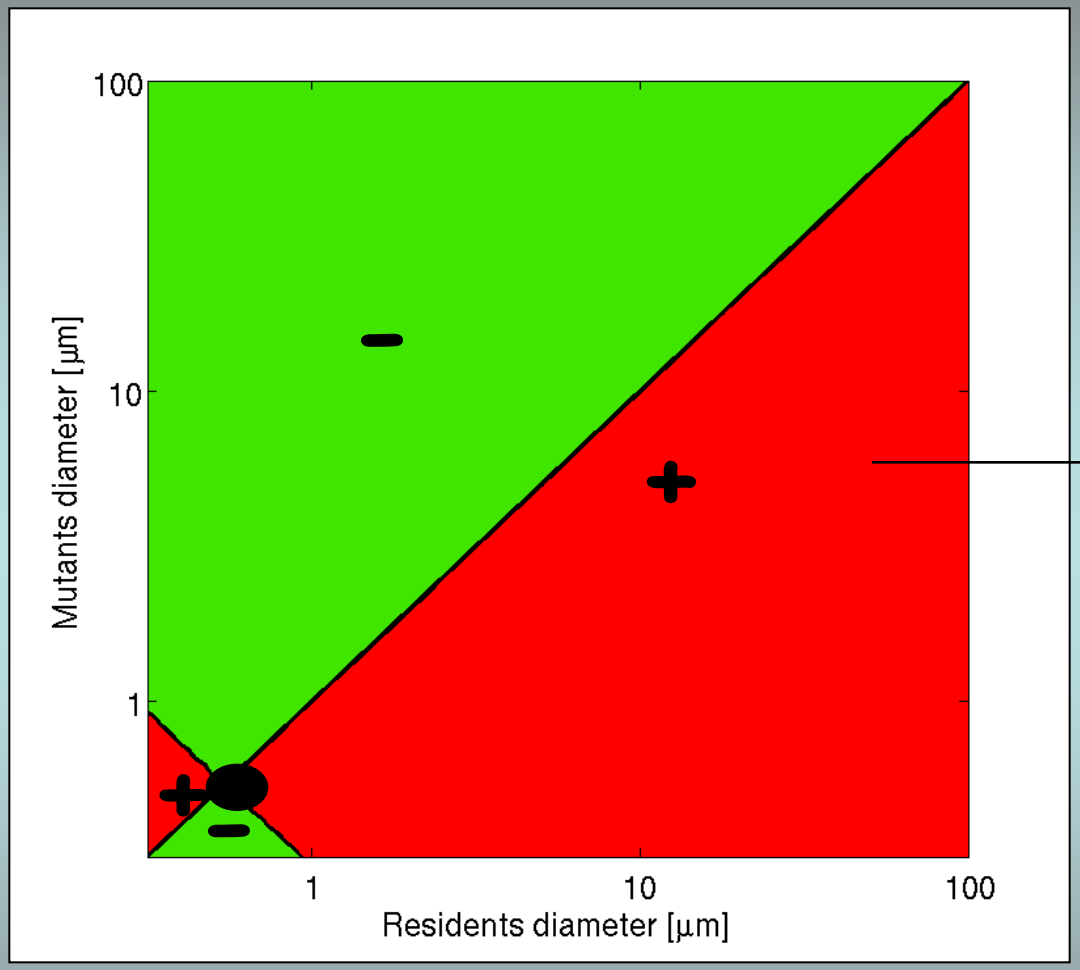


Growth rate limited by nutrient and light

Constant mortality → 0.01



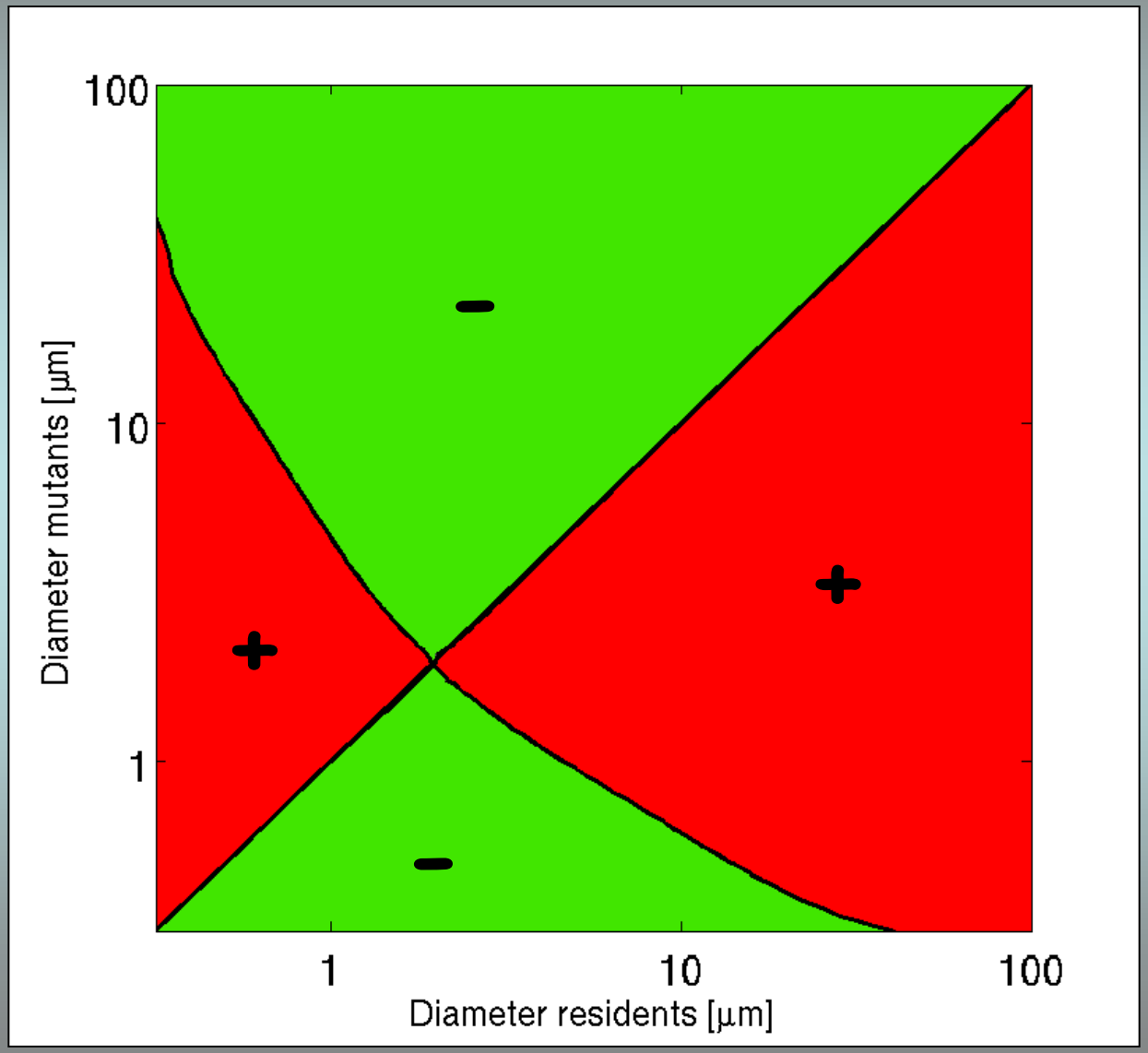
NP MODEL - Constant mortality



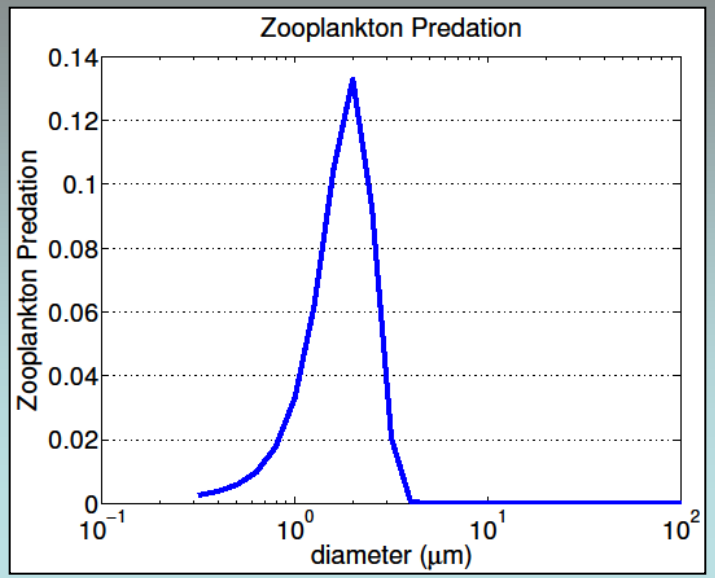
Mutant takes place of resident population

● Fixed Point

NP MODEL

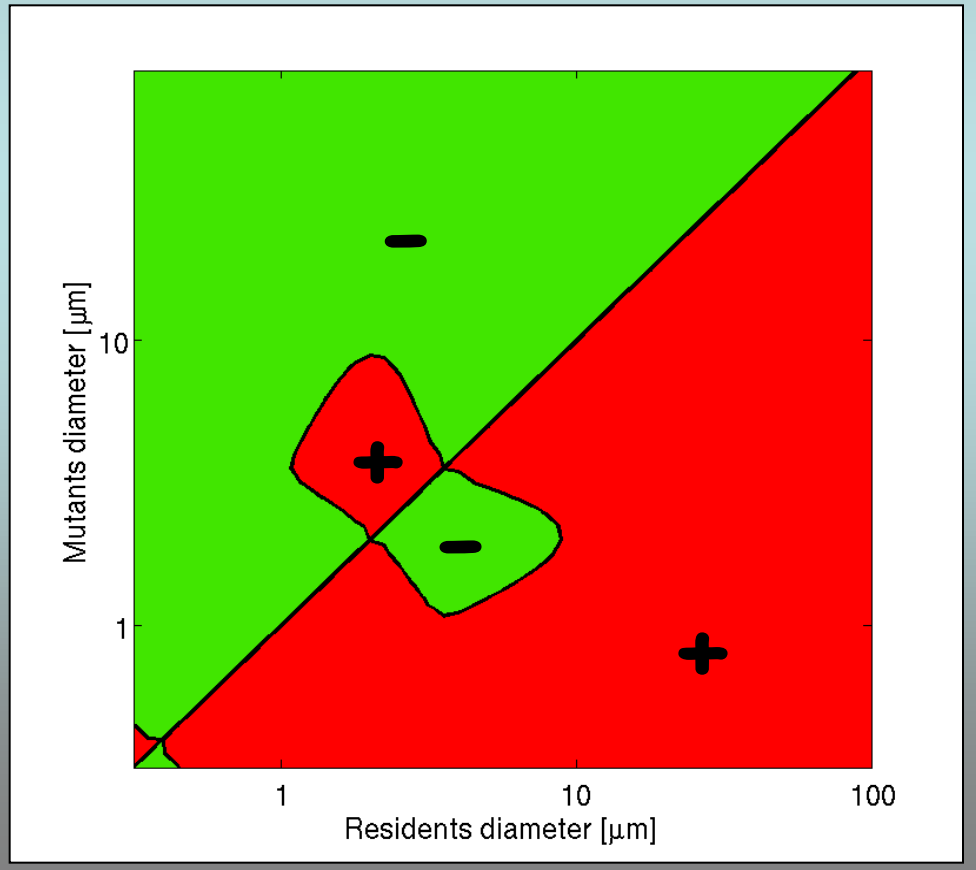


Predation depending on size

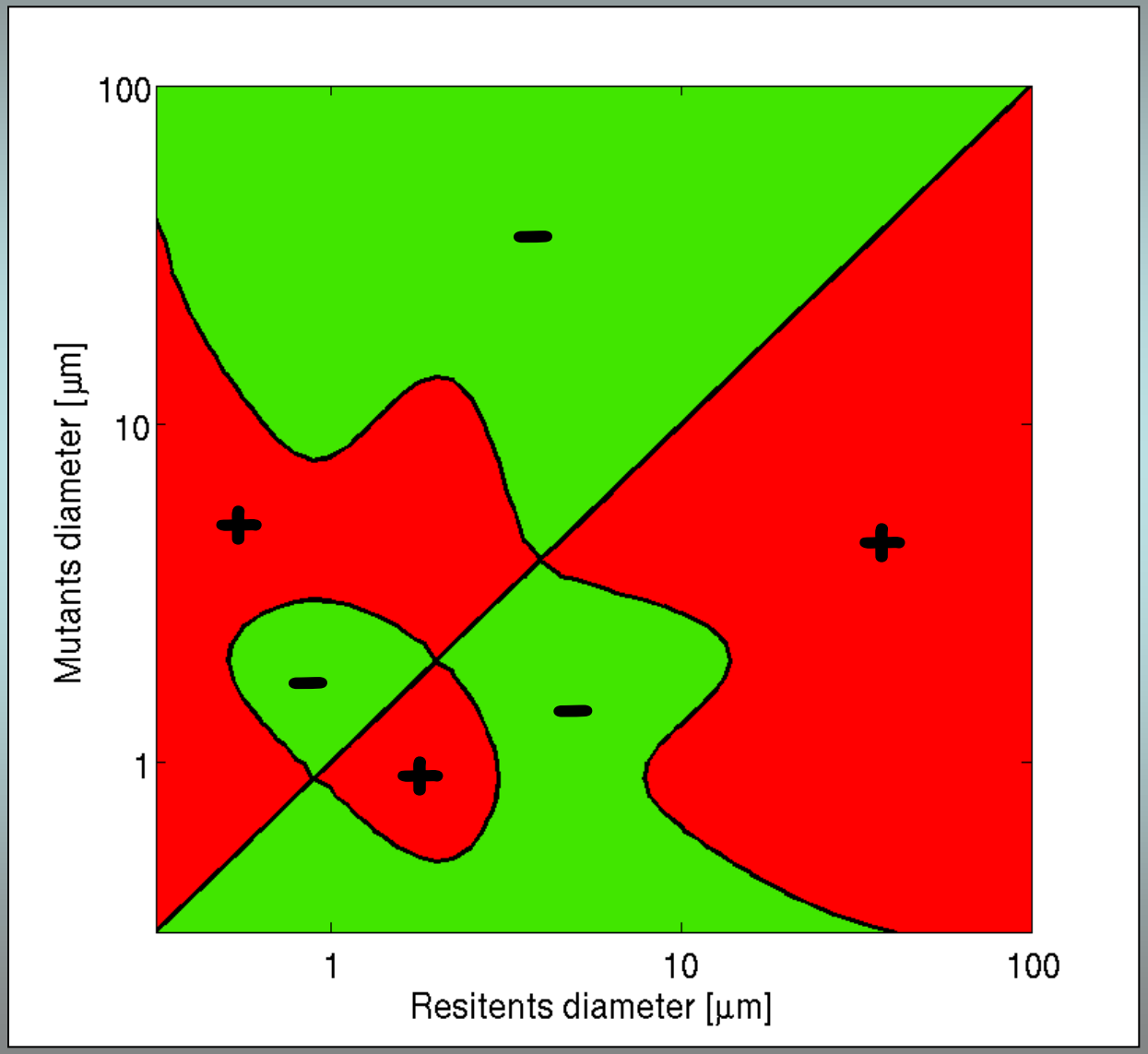


Constant mortality PIP

We introduce a zooplankton predation depending on size without a time dependent dynamics



Predation depending on size



NPZ MODEL

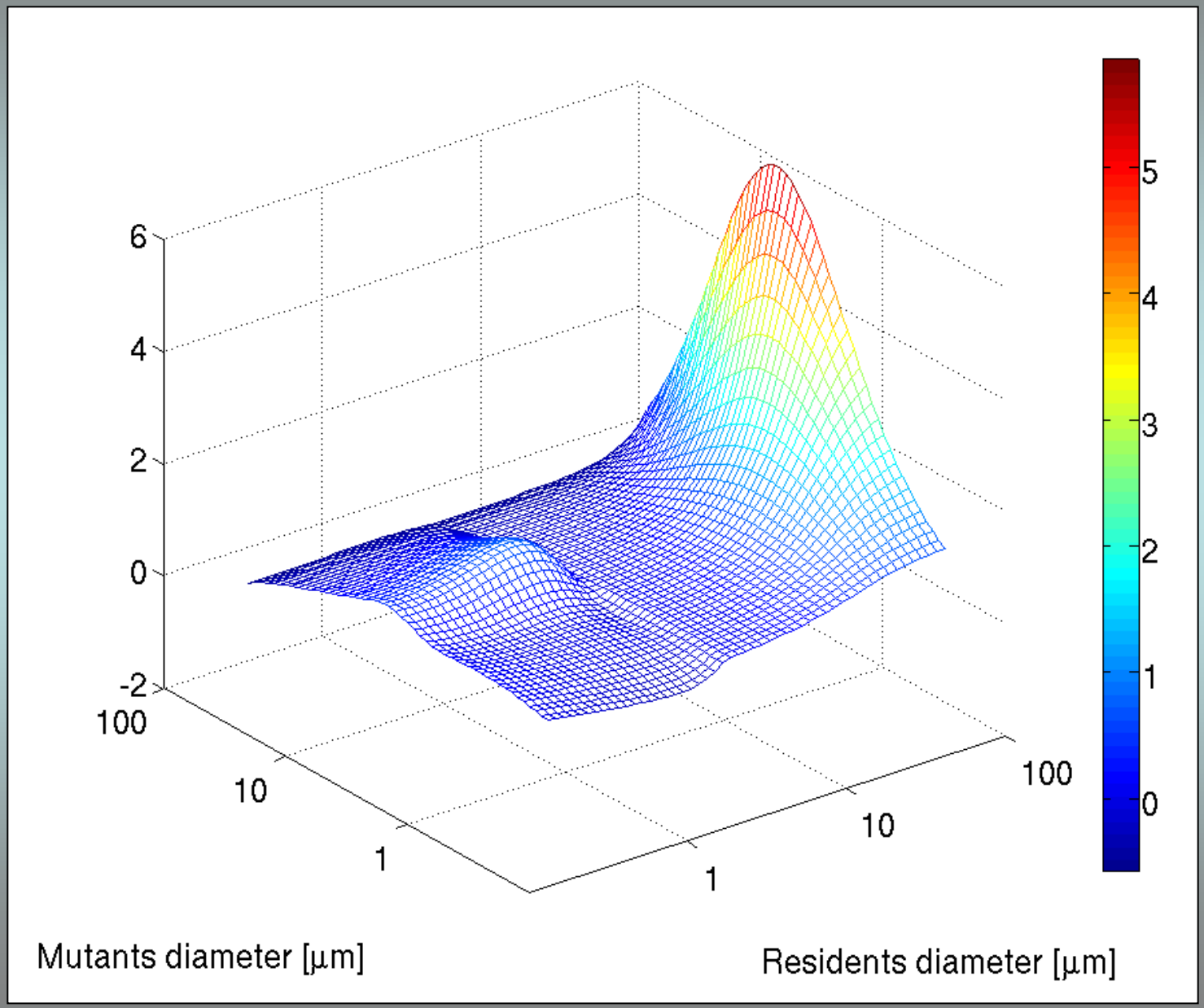
$$\begin{cases} \frac{\mathrm{d}N}{\mathrm{d}t} = I - g(N,L)\mu(x)P \\ \frac{\mathrm{d}P}{\mathrm{d}t} = [g(N,L)\mu(x) - m - s]P - g(P)C(x)Z \\ \frac{\mathrm{d}Z}{\mathrm{d}t} = [eC(x)g(P) - mz]Z \end{cases}$$

Convertion efficency

Zooplankton consumption rate depending on phytoplankton size

Zooplankton mortality

NPZ MODEL


6
4
2
0
-2
100
10
1
1
10
100
5
4
3
2
1
0
Mutants diameter [µm]
Residents diameter [µm]

NPZ MODEL