Optimization theories for saline environments

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Motivations

- It has long been suggested that, at the leaf scale, natural selection may have operated to provide increasingly efficient means of controlling the tradeoffs between water vapor loss and carbon gains.

- How does salinity modify this picture?
Photosynthesis - Transpiration

Diagram showing the process of photosynthesis and transpiration.
### Structural modifications

<table>
<thead>
<tr>
<th>NaCl (molal)</th>
<th>0.0</th>
<th>0.05</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mesophyll thickness (µm)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bean</td>
<td>150</td>
<td>165</td>
<td>260</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cotton</td>
<td>209</td>
<td>256</td>
<td>329</td>
<td>373</td>
<td>422</td>
<td></td>
</tr>
<tr>
<td>Atriplex</td>
<td>210</td>
<td>--</td>
<td>210</td>
<td>212</td>
<td>260</td>
<td>340</td>
</tr>
</tbody>
</table>

![Diagram showing mesophyll thickness comparison between control and 75% swws](image1.png)

![Graph showing mesophyll thickness vs NaCl concentration](image2.png)
Optimization model (no mesophyll limitations)

Define the instantaneous:

\[ \text{Carbon Gain} = f_c \]
\[ \text{Water Loss} = f_e \approx a g_s D \]

OBJECTIVE FUNCTION

\[ f(g_s) = f_c(g_s) - \lambda f_e(g_s) \]

Leaves are autonomous and attempt to maximize their own carbon gain for a given amount of water loss.
Basic equations:

<table>
<thead>
<tr>
<th>Without Salinity</th>
<th>With Salinity</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Transport Equation:</strong></td>
<td></td>
</tr>
<tr>
<td>( f_c = g_s (C_a - C_i) )</td>
<td>( f_c = g_{\text{eff}} (c_a - c_c) ); ( g_{\text{eff}} = \frac{g_c g_m}{g_c + g_m} )</td>
</tr>
<tr>
<td><strong>Biochemical Demand:</strong></td>
<td></td>
</tr>
<tr>
<td>( f_c = \frac{\alpha_1 (C_i - \Gamma^*)}{C_i + \alpha_2} )</td>
<td>( C_i \rightarrow C_c )</td>
</tr>
<tr>
<td><strong>Optimality Rule:</strong></td>
<td></td>
</tr>
<tr>
<td>( \frac{\partial}{\partial g_s} \left( f_c (g_s) - \lambda f_e (g_s) \right) = 0 )</td>
<td></td>
</tr>
</tbody>
</table>

Three equations with 4 unknowns \( (f_c, C_c, g_s, g_m) \) – mathematically unclosed
Optimization models (linear form) With mesophyll conductance

Upon differentiating $f(g_c)$ and setting it to zero:

$$f_c = \frac{a_1 g_m (c_a - c_p)}{a_1 + g_m (a_2 + sc_a)} \left[ 1 - \sqrt{\frac{a\lambda D}{c_a - c_p}} \right]$$

$$g_c = \frac{a_1 g_m}{(a_1 + g_m (a_2 + sc_a))} \left( -1 + \sqrt{\frac{c_a - c_p}{a\lambda D}} \right)$$

$$\frac{c_i}{c_a} = \frac{c_p}{c_a} + \frac{(c_a - c_p)}{c_a} \left[ 1 - \sqrt{\frac{(a\lambda D)}{(c_a - c_p)}} \right]$$
Olives (Loreto et al., 2003)

\[
f_c = g_c (c_a - c_p) \left[ \sqrt{\frac{a \lambda D}{c_a - c_p}} \right] = g_c \left[ \sqrt{a \lambda D (c_a - c_p)} \right]
\]

\[
f_c = \frac{a_1 g_m (c_a - c_p)}{a_1 + g_m (a_2 + sc_a)} \left[ 1 - \frac{a \lambda D}{c_a - c_p} \right]
\]
Preliminary Results (2)

\[
\frac{c_i}{c_a} = \frac{c_p}{c_a} + \frac{(c_a - c_p)}{c_a} \left[ 1 - \sqrt{\frac{(a\lambda D)}{(c_a - c_p)}} \right]
\]

Bongi and Loreto, 1989

Olives

VPD=10 mbar
VPD=30 mbar

Ball and Farquhar, 1984

Mangroves

VPD=6 mbar
VPD=12 mbar
VPD=24 mbar
Preliminary conclusions

- 2 time scales of stomata response to increasing salinity:
  - FAST t.s.: stomatal conductance
  - SLOW t.s.: mesophyll conductance
- Optimization theory consistent with datasets analyzed
- Three parameters were used to analyze salt tolerance by different species (olives-mangroves):
  - $g_m$: changes in all species
  - $\lambda$: changes only for salt-intolerant species
  - $a_1$: changes for the olive case
Further work

- Short Term: literature review to assess how salinity affects the optimization theory parameters for different species

- Long Term: integrate this results with on-going work on hydrologic models of the soil-plant system to assess how climate change (e.g. CO2, VPD, T) affects plant productivity in salt environments
References