Energy of River Deltas

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River Deltas

- **goals:**
  - explain different river delta forms from mass- and momentum balance considerations
- **assumptions:**
  - steady-state deltas: sediment input = sediment export
  - neglect tidal influences

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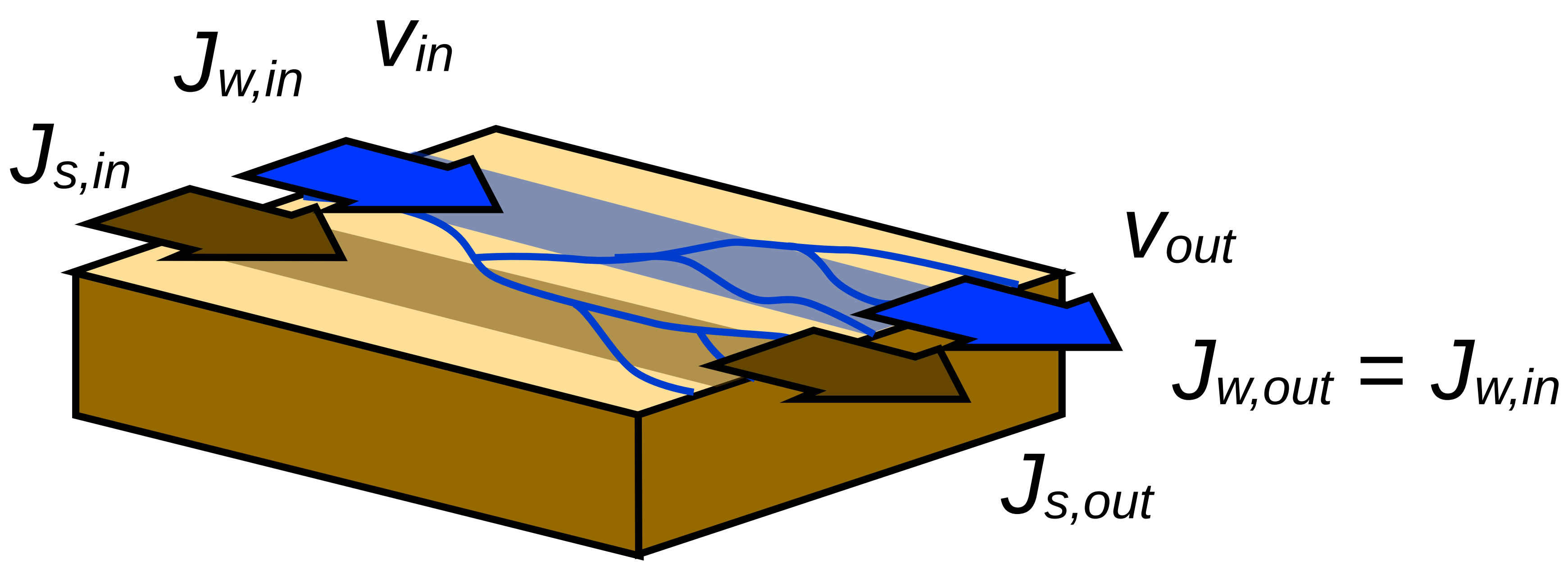
three ingredients:

- momentum balance of water flow p_w
- work done to detach sediment $P_{w,s}$
- mass balance of suspended sediment m_s



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momentum balance (water):

$$\frac{dp_w}{dt} = J_{w,in}v_{in} - F_{w,d} - J_{w,in}v_{out} = 0$$

$$v_{out} = \frac{J_{w,in}v_{in} - F_{w,d}}{J_{w,in}}$$

sediment detachment:

$$P_{w,s} = (F_{w,s} - F_{crit})v_{out}$$

mass balance (sediments in suspension):

$$\frac{dm_s}{dt} = J_{s,in} + \mu P_{w,s} - \frac{m_s}{\tau_s} - \frac{m_s v_{out}}{L} = 0$$

$$m_s = (J_{s,in} + \mu P_{w,s})\frac{\tau_s L}{L + v_{out}\tau_s}$$

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sediment export:

$$J_{s,out} = \frac{m_s v_{out}}{L} = (J_{s,in} + \mu P_{w,s}) \frac{v_{out}\tau_s}{L + v_{out}\tau_s}$$

dimensionless number N_s:

$$N_s = \frac{L/\tau_s}{v_{out}}$$

*characterizes the ratio of **sediment deposition** vs. **sediment export***

$$J_{s,out} = \frac{(J_{s,in} + \mu P_{w,s})}{1 + N_s}$$

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limit of small N_S:

$$J_{s,out} \approx J_{s,in} + \mu P_{w,s}$$

limit of large N_S:

$$J_{s,out} \approx \frac{(J_{s,in} + \mu P_{w,s})}{L/\tau_s} v_{out}$$

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- we look for theoretical steady state deltas for which:

$$J_{s,in} = J_{s,out}$$

- that implies that deposition equals work to bring sediments into suspension:

$$\mu P_{w,s} = \frac{m_s}{\tau_s}$$

$$\mu P_{w,s} = (J_{s,in} + \mu P_{w,s}) \frac{1}{1 + v_{out}\tau_s/L}$$

$$\mu P_{w,s} = J_{s,in} N_s$$

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$$\mu P_{w.s} = J_{s.in} N_s$$

- **Small Ns = export > deposition**
  - little deposition => less work needed to entrain new grains to maintain steady state
  - less work => little drag => small wetted perimeter => fewer channels
  - achieved by high momentum influx or low sediment influx

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$$\mu P_{w.s} = J_{s.in} N_s$$

- **Large Ns = deposition > export**
  - high deposition => need to perform more work to entrain new grains to maintain steady state
  - high work => high drag => high wetted perimeter => many channels (branching)
  - achieved by low momentum import or high sediment influx

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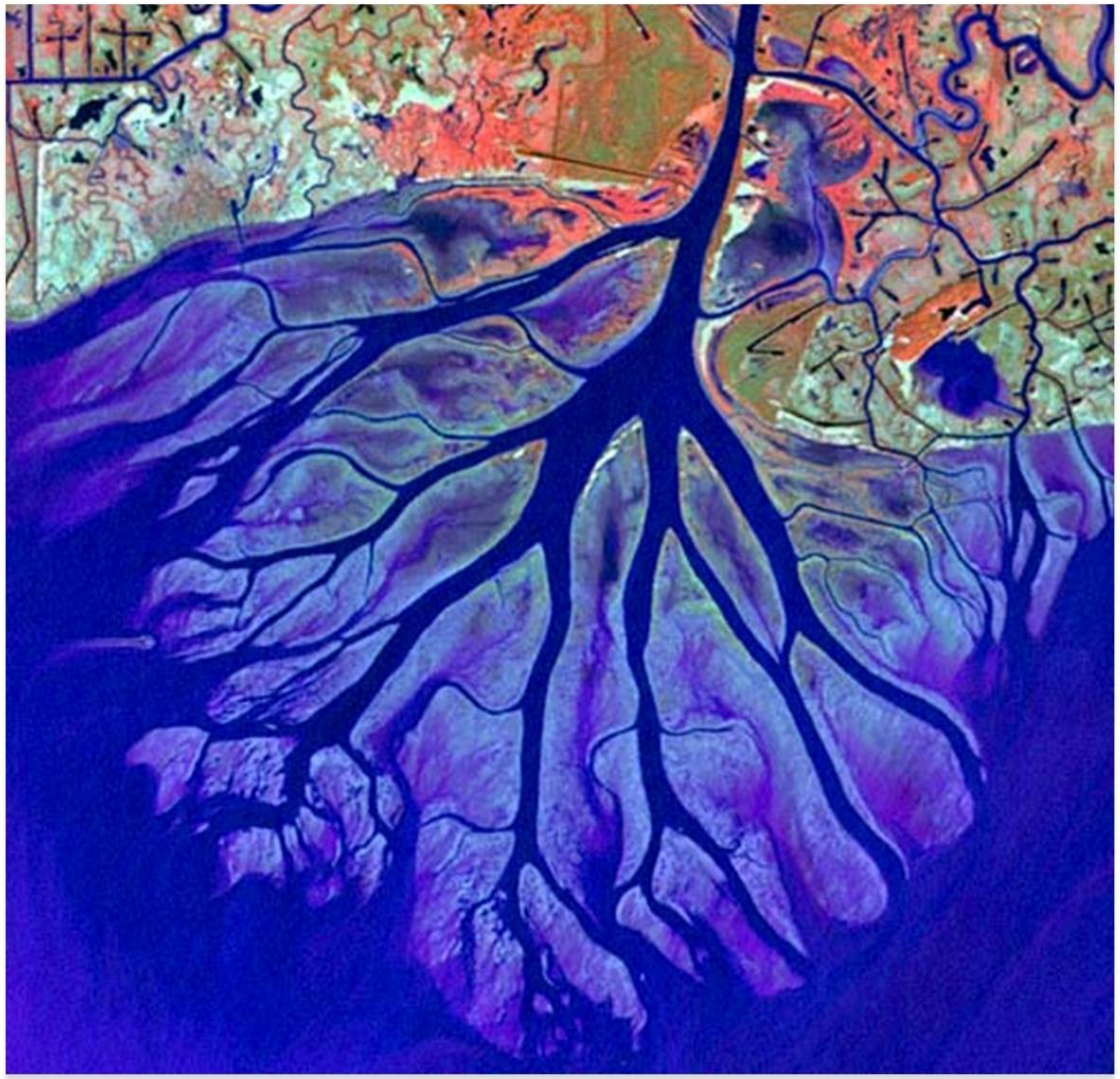
- Large Ns - lots of work
- Wax Lake Delta, Louisiana



Satellite image, courtesy of Louisiana State University

- Small Ns - less work
- Mississippi Bird's Foot, Louisiana

Google Earth image

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- Possible next steps:
  - Calculate drag and relate to wetted perimeter to determine number of channels
  - Tides?
  - Look at non-steady state systems and use the energy balance to describe formative delta dynamics